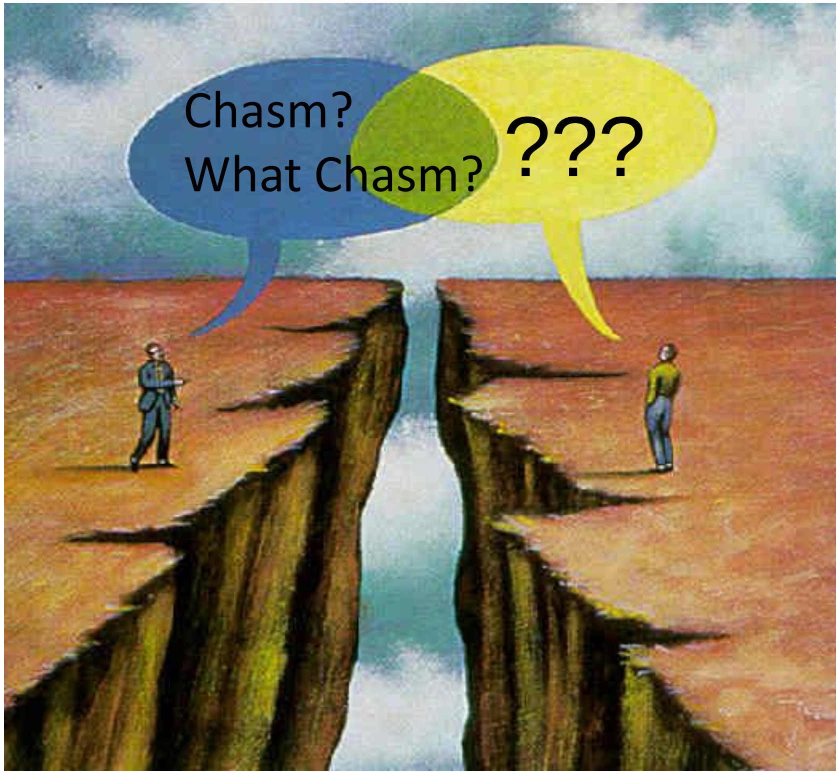
What we say/What they hear

Culture shock in the classroom



Culture is, by its very nature, completely **UNCONSCIOUS**

Cultural Elements

- We hold presuppositions and assumptions that are unlikely to be shared by a student who is new to mathematical culture.
- We know where to focus of our attention and what can be safely ignored.
- We have skills and practices that make it easier to function in our mathematical culture.

“That’s obvious”

To a mathematician this means “this can easily be deduced from previously established facts.”

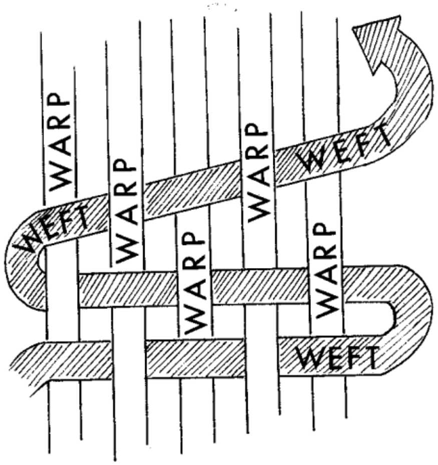
Many of my students will say that something they already “know” is “obvious.”

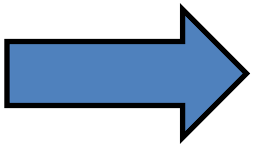
For instance, if I give them the field axioms, and then ask them to prove that

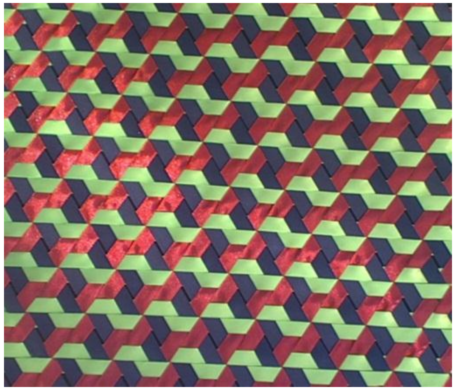
$$\text{for all } x \in F, x \cdot 0 = 0$$

they are very likely to wonder why I am asking them to prove this, since it is “obvious.”

The Purpose of Proof



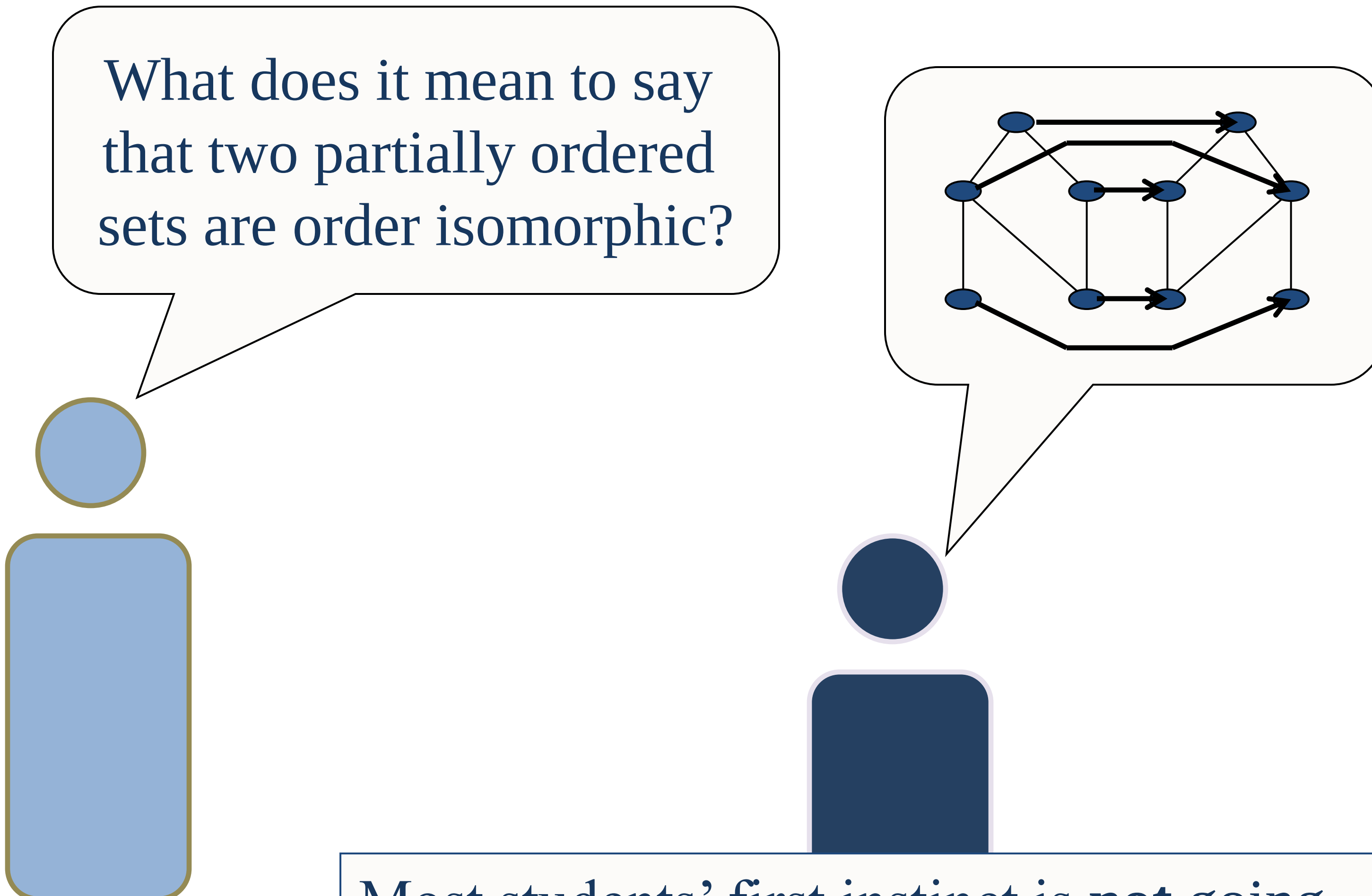




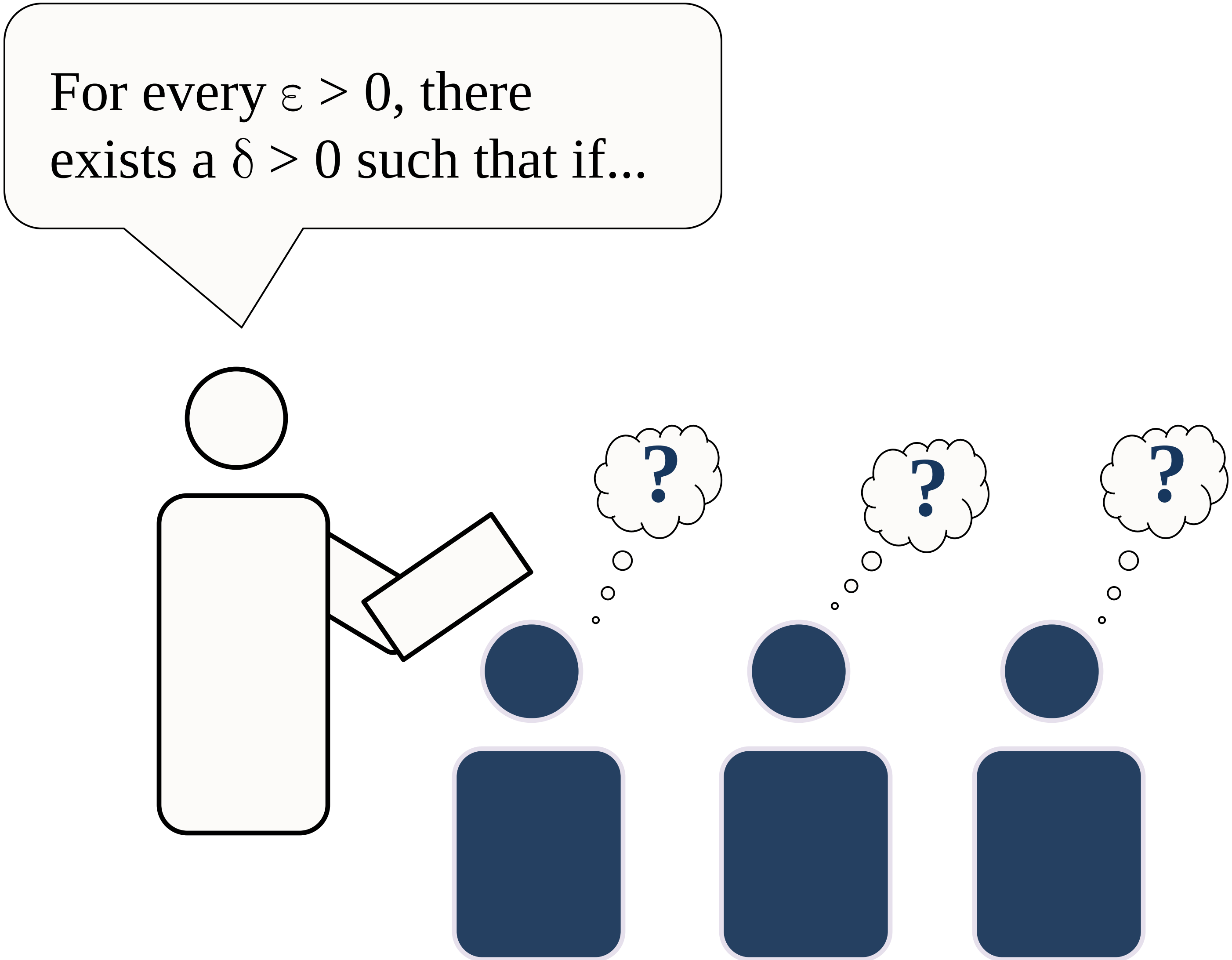
Sometimes proofs help us understand connections between mathematical ideas.

Presuppositions and Assumptions

What is the definition of Definition?



Most students’ first instinct is **not** going to be to say that there exists an order-preserving bijection between them!


For every ε > 0, there exists a δ > 0 such that if...
?
?
?

As if this were not bad enough, we mathematicians sometimes do some very weird things with definitions.

Definition: Let Ω be a collection of non-empty sets. We say that the elements of Ω are **pairwise disjoint** if given A, B in Ω, either $A \cap B = \emptyset$ or $A = B$.

WHY NOT....

Definition: Let Ω be a collection of non-empty sets. We say that the elements of Ω are **pairwise disjoint** if given any two distinct elements A, B in Ω, $A \cap B = \emptyset$.

???

Mathematical Culture

We know where to

focus our attention

for maximum benefit

and we know

what can be safely ignored.

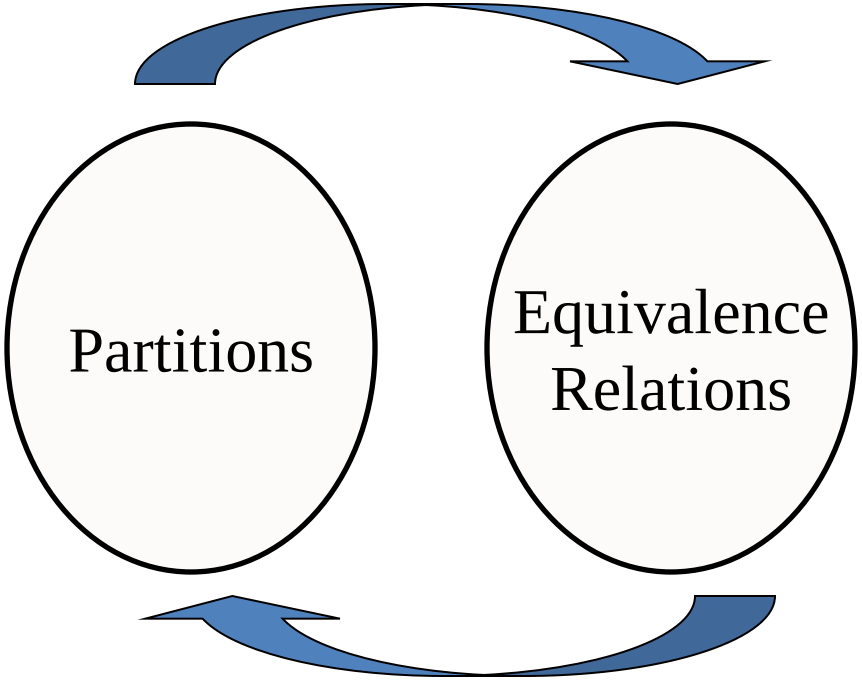
What? . . . Where?

Helping our students focus

Example: Equivalence Relations

We want our students to understand the duality between partitions and equivalence relations.

We want them to prove that every equivalence relation naturally leads to a partitioning of the set, and vice versa.

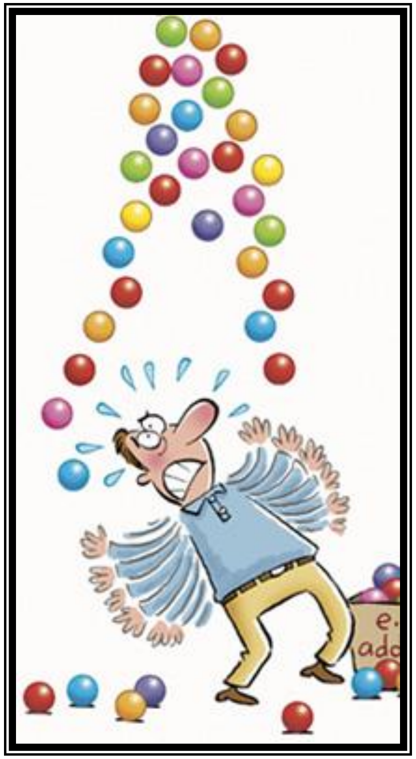


Us

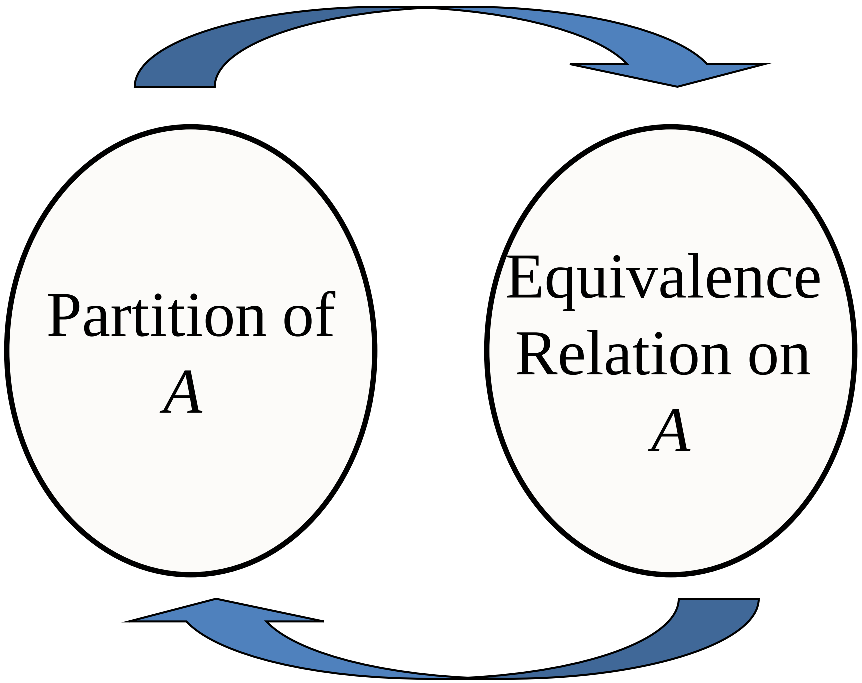
There is a lot going on in this theorem.

Many of our students are completely overwhelmed.

Our students!



Sorting out the Issues

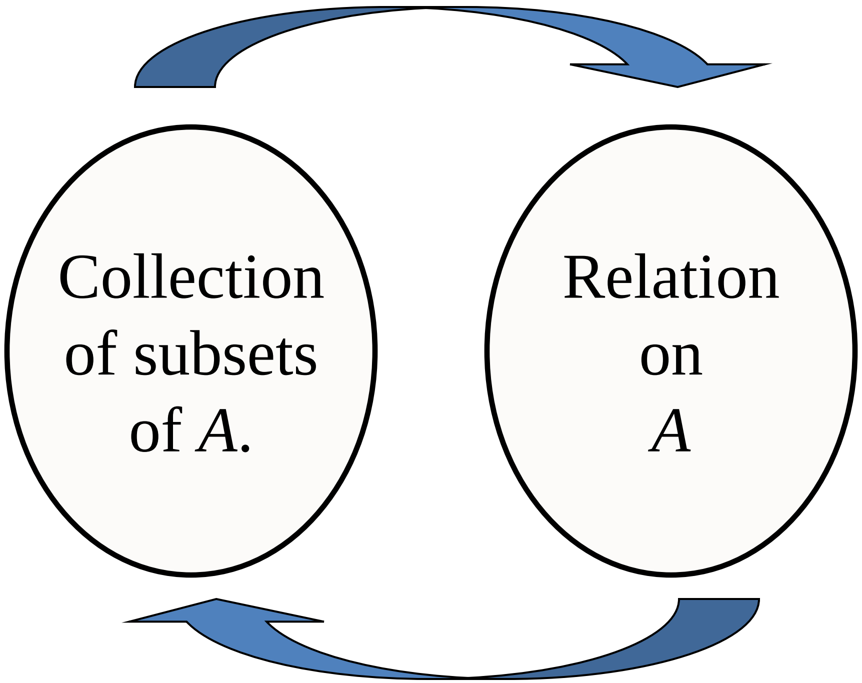


Every partition of a set *A* generates an equivalence relation on *A*.

&

Every equivalence on *A* relation generates a partition of *A*.

Sorting out the Issues



Every collection of subsets of *A* generates a relation on *A*.

&

Every relation on *A* generates a collection of subsets of *A*.

And. . .It’s not just about logical connections

The usual practice is to define an equivalence relation first and only then to talk about partitions.

$\boxed{\text{Are we directing our students' attention in the } \textit{wrong} \text{ direction?}}$

Scenario: You have just defined subspace (of a vector space) in your linear algebra class:

Definition: Let V be a vector space. A subset S of V is called a subspace of V if S is closed under vector addition and scalar multiplication.

The obvious thing to do is to try to see what the definition means in $\mathbb{R}^2$ and $\mathbb{R}^3$. You could show your students, but you would rather let them play with the definition and discover the ideas themselves.


. . . closed under **scalar multiplication**
and closed under **vector addition** . . .

Mathematical Culture

We have

skills and practices

that make it easier to function in our mathematical culture.

First Definitions

In beginning real analysis, we typically begin with sequence convergence.

Given any tolerance

Definition: $a_n \to L$ means that for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n > N$, $d(a_n, L) < \varepsilon$.

there is some fixed position

beyond which

a_n is within that tolerance of *L*

First Definitions

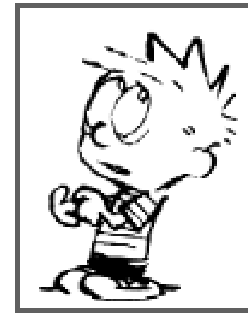
In beginning real analysis, we typically begin with sequence convergence:

Definition: $a_n \to L$ means that for every $\varepsilon > 0$,

there exists $N \in \mathbb{N}$ such that for all $n > N$, $d(a_n, L) < \varepsilon$.

What does all this have to do with Pedagogy?

Don’t just stand there!
Do something.



- $a_n \to L$ means that for all $\varepsilon > 0$ there exists $n \in \mathbb{N}$ such that $d(a_n, L) < \varepsilon$.
- $a_n \to L$ means that for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for some $n > N$, $d(a_n, L) < \varepsilon$.
- $a_n \to L$ means that for all $N \in \mathbb{N}$, there exists $\varepsilon > 0$ such that for all $n > N$, $d(a_n, L) < \varepsilon$.
- $a_n \to L$ means that for all $N \in \mathbb{N}$ and for all $\varepsilon > 0$, there exists $n > N$ such that $d(a_n, L) < \varepsilon$.

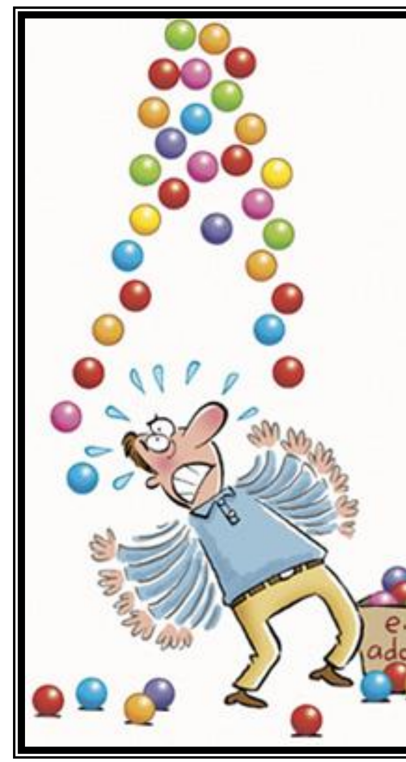
Students are asked to think of these as “alternatives” to the definition. Then they are challenged to come up with examples of real number sequences and limits that satisfy the “alternate” definitions but for which $a_n \to L$ *is false*.

Us

Lectures are linear and orderly.

Active mathematical thinking typically is not.

Our students!



Large-scale study

comparing IBL and non-IBL courses (Laursen et. al.)

found:

Strong and consistent evidence about the dual importance of individual engagement and collaborative learning processes" for student learning outcomes.

Laursen et. al

Statistically significant results: student learning outcomes are

- correlated with the *fraction of class time spent doing student-centered activities*
- and anti-correlated with the *fraction of time spent listening to instructors talk.*

Similar positive and negative correlations were seen with the proportion of class time that was student- or instructor-led.

Laursen, S., Hassi, M.-L., Kogan, M., Hunter, A.-B., & Weston, T. (2011). *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*. [Report prepared for the Educational Advancement Foundation and the IBL Mathematics Centers]. Boulder, CO: Ethnography & Evaluation Research, University of Colorado Boulder.

The full report can be found at http://www.colorado.edu/eer/research/documents/IBLmathReportALL_050211.pdf